NASA TN D-115

NASA TN D-115

7N-18
197136
288

TECHNICAL NOTE

D-115

PREDICTED CHARACTERISTICS OF AN INFLATABLE ALUMINIZED-PLASTIC SPHERICAL EARTH SATELLITE WITH REGARD TO TEMPERATURE, VISIBILITY, REFLECTION OF RADAR WAVES, AND PROTECTION FROM ULTRAVIOLET RADIATION

By George P. Wood and Arlen F. Carter

Langley Research Center
Langley Field, Va.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

WASHINGTON October 1959

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

TECHNICAL NOTE D-115

PREDICTED CHARACTERISTICS OF AN INFLATABLE ALUMINIZED-PLASTIC SPHERICAL EARTH SATELLITE WITH REGARD TO TEMPERATURE, VISIBILITY, REFLECTION OF RADAR WAVES, AND PROTECTION FROM ULTRAVIOLET RADIATION

By George P. Wood and Arlen F. Carter

SUMMARY

An investigation was made to predict whether a hollow aluminized-plastic sphere in a terrestrial orbit at an altitude of about 1,000 miles would be easily visible to the naked eye, would be a good reflector of radar waves, would be protected from deterioration of the plastic by ultraviolet radiation, and would assume acceptable extremes of temperature. It was found that a vapor-deposited aluminum coating with a thickness of 2,200 angstroms on 1/4-mil-thick Mylar forming a 100-foot-diameter sphere would probably meet each of the requirements regarding radiation.

INTRODUCTION

Among the satellites that have been proposed is one constructed as a large, lightweight, inflatable, plastic spherical shell - in other words, a balloon. (This and other pneumatically erectable configurations have been proposed by W. J. O'Sullivan of the Langley Research Center.) It would be desirable for this satellite to be easily visible to the naked eye, to be a good reflector of radar signals, and to have a long lifetime in orbit. The satellite would then have a number of uses which need not be discussed herein.

In addition to requirements of maximum weight and approximate minimum size, four requirements involving electromagnetic radiation in the spectrum from the extreme ultraviolet to radar wavelengths, inclusive, were desired to be met. These requirements were:

(1) that the satellite reflect sufficient visible radiation from the sun to be readily discernible by the naked eye when at an altitude of the order of 1,000 miles

(2) that the satellite have a high reflectivity for radar waves

(3) that the plastic that formed the shell of the satellite be protected from deterioration by long-term exposure to the sun's ultraviolet radiation, and

(4) that the satellite assume, by virtue of its reflectivity to solar radiation and its emissivity in the infrared region, an equilibrium temperature in sunlight that was not too high and in the earth's shadow that was not too low.

The purpose of the present paper is to describe the methods used and the results obtained in designing the vehicle so that it would, insofar as was practicable, meet the four requirements involving radiation.

Measurements of the reflectivity of the satellite material to simulated sunlight were made for the National Aeronautics and Space Administration through the courtesy of G. Hass of the U.S. Army Engineer Research and Development Laboratories and of I. Nimeroff of the National Bureau of Standards. Measurements of the reflectivity for radar waves of the aluminum skin of the satellite were made through the courtesy of D. E. Dustin of the M.I.T. Lincoln Laboratory and of J. R. Pierce, Director of Electronic Research, Bell Telephone Laboratories, Inc. Measurements of the emissivity of the aluminum (outside) surface and of the Mylar (inside) surface of the satellite were made by L. F. Drummeter and E. Goldstein of the U.S. Naval Research Laboratory and by T. O. Thostesen of the Jet Propulsion Laboratory.

SYMBOLS

a	albedo of earth
A	absorptivity (fraction of incident radiant energy which is absorbed)
c	velocity of light
C_S	solar constant
D	diameter
G	gain of antenna
h	altitude

I	intensity of radiation
$k = \dfrac{r_E}{r_E + h}$	
l_R	distance between receiving antenna and satellite
l_T	distance between transmitting antenna and satellite
m	stellar magnitude
P	power
l	distance
R	reflectivity (fraction of incident radiant energy which is reflected)
r	radius
S	surface area
t	thickness
T	temperature
α	absorption coefficient
ϵ	emissivity
θ	angle of incidence
κ	extinction coefficient
λ	wavelength
μ	permeability
σ	electrical conductivity or Stefan-Boltzmann constant, 8.135×10^{-11} cal$/(\mathrm{cm}^2)(\mathrm{min})(^\circ\mathrm{K}^4)$
ϕ	angle of reflection or emission

Subscripts:

E	earth, earth's radiation

i	inside
o	outside
0	initial
R	received, receiver, or receiving
S	solar, solar radiation
sat	satellite
T	transmitted, transmitter, or transmitting
CS	coldest spot
HS	hottest spot

L 2 5 2

GENERAL CONSIDERATIONS

The spherical shape was chosen in order to provide a configuration that would not scintillate if the vehicle rotated or tumbled, inasmuch as scintillation was undesirable for a radar reflector, and to provide a configuration for which there was no difficult orientation problem, as there would be for a plane mirror. To meet the weight limit, which was approximately 100 pounds including the container, and at the same time to provide as large an object as possible, 1/4-mil (0.00025 inch) plastic film was chosen to form a sphere 100 feet in diameter. The sphere can be placed into orbit in a folded condition and then inflated by approximately 1 pound of gas contained at high pressure in a small tank or by the vapor pressure of a liquid. The plastic would weigh about 60 pounds, and some margin would remain for the weight of seams, the container, and the inflating gas or vapor. Clear plastic does not meet the first three requirements regarding radiation. Pigmented plastic of a light color, if it were available, would meet the visibility and thermal requirements but not the others. Plastic coated with aluminum paint would meet the four radiation requirements, but the weight would be too great. Accordingly, a thin film of vapor-deposited aluminum on the outside of the satellite was chosen as probably the best practical solution to the problem.

It may be pointed out here, as is stated in reference 1, that the best method of obtaining a thin film of aluminum is to deposit the film by evaporation, and not by sputtering or any other method, that a high vacuum ($\leqq 10^{-4}$ mm Hg) should be used, that the vapor should arrive at the condensing surface reasonably close to normal incidence, or at least certainly not at an angle approaching grazing incidence, and that the film should be deposited rapidly, in the order of 10 seconds or less.

REFLECTION OF VISIBLE RADIATION

The initial choice of thickness for the aluminum film was made on the basis of data on the reflectivity in the visible portion of the spectrum of carefully prepared aluminum films. The data that were used were some experimental results published by Holland (ref. 2) which are shown replotted in figure 1. This figure shows the reflectivity and the transmission for radiation of a wavelength of 4,600 A (angstrom) as a function of thickness of film and is considered applicable to visible solar radiation because solar radiation has a rather narrow and high maximum at about 4,700 A. At a thickness of 375 A the transmission is very nearly zero and the reflectivity is 0.9.

These data were, of course, obtained on films that were produced in the laboratory by a very carefully controlled process and that were very clean. To provide a margin of safety, because the material would be handled during assembly and packaging, and to allow for nonuniformities in film thickness, the thickness chosen for use on the satellite was nearly six times 375 A, or 2,200 A (about 9 microinches).

In reference 2 it was also reported that the thickness of the film calculated from the loss of mass of the source of aluminum vapor agreed within 5 percent with the thickness measured by the multiple-beam interferometric method of Tolansky if the density of the film was taken to be equal to that of the bulk metal. Thus, a 2,200-A-thick coating of aluminum on a 100-foot-diameter sphere would have a weight of 4 pounds.

Samples of the satellite material were prepared by a contractor and the reflectivity of the material to simulated sunlight was measured and found to be 0.90.

The question, then, is what will be the visual magnitude of a 100-foot aluminized spherical satellite at an altitude of 1,000 miles and illuminated with sunlight. The satellite will intercept sunlight by its projected area and will specularly reflect uniformly in all directions (ref. 3, for example). The sun's brightness will therefore be attenuated in the ratio of the product of the projected area of the satellite and its reflectivity, to the surface area of a sphere of 1,000-mile radius. The sun's brightness is, therefore, attenuated by a factor of 2.02×10^{-11}. The apparent visual brightness of the sun expressed as stellar magnitude is -26.7. Since each unit change in magnitude changes the brightness by a factor of $(100)^{1/5}$ or 2.512,

$$(2.512)^{(-26.7-m)} = 2.02 \times 10^{-11}$$

where m is the visual magnitude of the satellite. From this equation, $m = 0.04$. The satellite would therefore appear as a rather bright object. For comparison, the brightest stars and their magnitudes (ref. 4) are as follows:

Star	Magnitude, m
Sirius	-1.58
Canopus	-.86
α Centauri	.06
Vega	.14
Capella	.21
Arcturus	.24

L252

The satellite would therefore be brighter than all but two stars, Sirius in the northern hemisphere and Canopus in the southern hemisphere.

If the 100-foot sphere were a satellite of the moon, its magnitude would be 11.9. The minimum telescope for observing it would be one having a diameter of about 4 inches (ref. 4). Such a small telescope, however, could probably be used successfully only when a minimum portion of the moon's disk was illuminated.

REFLECTION OF RADAR WAVES

It is well known that, as the wavelength of the incident radiation increases, the reflectivity of bulk metal generally increases and closely approaches unity in the midinfrared region where reflection depends on only the free and not the bound electrons. (Aluminum has a slight dip in reflectivity between about 4,000 A and 20,000 A.) A metal can therefore be expected to reflect nearly 100 percent in the radar region. This is true also for a metallic film despite the fact that, although the thickness of the metal may be of the order of a wavelength in the infrared region, it will be about five orders of magnitude smaller than a wavelength in the radar region. This point is mentioned because a number of the people who were interested in the use of this proposed satellite as a reflector of radar signals felt that the metal thickness was too small for the absorption to be significant and that therefore the signals would not be reflected. It is true that the absorption is not large. Large absorption, however, is not necessary for high reflectivity; only a large extinction coefficient is necessary. It can be shown from the boundary conditions which the electric and the magnetic vectors must satisfy, as deduced from Maxwell's equations, that, provided λ and σ are large enough (as they are in this case), the reflectivity of a metal is given by (ref. 5)

$$R \approx 1 - \frac{2}{(\mu\sigma\lambda c)^{1/2}}$$

For the film in question, the direct-current conductivity was measured as 4×10^6 mho/m. (This is about one-ninth the value for the bulk metal.) For the values

$$\sigma = 4 \times 10^6 \text{ mho/m} \quad \left(\text{or } \frac{\text{amp}}{\text{volt-m}}\right)$$

$$\mu = 4\pi \times 10^{-7} \frac{\text{volt-sec}}{\text{amp-m}}$$

$$\lambda = 0.1 \text{ m}$$

$$c = 3 \times 10^8 \text{ m/sec}$$

the reflectivity should be

$$R \approx 1 - \frac{2}{12,300} \approx 1$$

This result was substantiated by measurements, as follows:

Source	Frequency, Mc/sec	Reflectivity, R
M.I.T. Lincoln Laboratory	2,880	0.98
Bell Telephone Laboratories	4,000	>0.98
Bell Telephone Laboratories	11,000	>0.98
Langley Research Center	20,000	0.97

The aluminum skin of the satellite is therefore a good reflector of radar waves. A sphere, however, is not a very good configuration for a reflector. The minimum transmitter power required for receiving intelligible signals reflected from the satellite depends of course on the kind of information that is to be transmitted, but the approximate transmitter power required for very simple systems and for systems of modulation that require much greater signal-to-noise ratios can be calculated. The ratio of the power received to the power transmitted is (ref. 6, eq. 3(a))

$$\frac{P_R}{P_T} = \frac{G_T}{4\pi l_T^2} \frac{\frac{\pi}{4}(D_{sat})^2 R_{sat}}{4\pi l_R^2} G_R \frac{\lambda^2}{4\pi}$$

The antenna gain G is

$$G = \frac{4\pi(0.6A_a)}{\lambda^2}$$

where A_a is the area of the aperture of the antenna. Therefore,

$$\frac{P_R}{P_T} = \frac{0.36A_{a,T}A_{a,R} \frac{\pi}{4}(D_{sat})^2 R_{sat}}{4\pi(l_T l_R \lambda)^2}$$

If the diameters of the transmitting and the receiving antennas are 50 feet, the wavelength is 10 centimeters, the diameter of the satellite is 100 feet, l_T is 1,000 miles, and l_R is 3,000 miles, then

$$\frac{P_R}{P_T} \approx 10^{-18}$$

or P_R is down 180 decibels from P_T. If the minimum power required at the receiver is taken to be 100 decibels below 1 milliwatt, then a minimum of 100 kilowatts of transmitter power is required. If a minimum power required at the receiver 70 decibels below 1 milliwatt is assumed, a minimum of 100 megawatts of transmitter power is required. These two transmitter powers fairly well bracket the range of the order of magnitude of power that might be needed for successfully using the satellite as a reflector. (A more extensive discussion of some of the problems involved in using a satellite as a microwave relay station is given in ref. 7.)

TRANSMISSION OF ULTRAVIOLET RADIATION

It is well known that the portion of the sun's ultraviolet radiation that penetrates the earth's atmosphere has a deteriorating effect on Mylar; it drastically degrades the tensile strength and integrity of Mylar. It was therefore thought to be advisable to protect the structure of the satellite from the sun's ultraviolet radiation, which is more intense outside the earth's atmosphere than at the earth's surface.

The ultraviolet region can be taken as the region from 4,000 A to about 300 A. For convenience in discussion here, it is divided into three regions.

Ultraviolet Region Between 4,000 A and 2,200 A

The reflectivity of both a mirror coating of aluminum and of a film about 600 A thick is given in reference 8 as about 0.92 at wavelengths from 4,000 A to 2,200 A if the coating is deposited under ideal conditions and if the reflectance is measured immediately after deposition of the coating. It is also shown that the reflectivity was decreased by less than 2 percent by aging for a year in normal air with 30 to 60 percent humidity. Furthermore, the reflectivity was decreased only 2 percent by exposure for 68 hours to the ultraviolet radiation of a 300-watt quartz-mercury lamp at a distance of 6 inches.

Data on the extinction coefficient κ could be found in the literature only for a wavelength of 4,000 A. At this wavelength κ is given as 3.92 in reference 9. Therefore, the fraction transmitted by a film 2,200 A thick is

$$I/I_0 = e^{-4\pi\kappa t/\lambda} \approx 10^{-12}$$

Since I_0 is 0.08 of the incident radiation (0.92 is reflected), the transmitted radiation is only 10^{-13} of the incident radiation and therefore is much too small to be of any significance.

Ultraviolet Region Between 2,200 A and 900 A

The reflectivity of evaporated aluminum films in the vacuum ultraviolet region between 2,200 A and 900 A was reported in reference 10. High speed of evaporation was found to be the most important factor in producing aluminum films of highest ultraviolet reflectivity. In this region as in others, the greatest reflectivity was obtained when the film was deposited in a few seconds. Film thicknesses between 500 A and 2,000 A had very nearly the same reflectivity, which was 0.90 at 2,000 A, 0.74 at 1,600 A, 0.38 at 1,200 A, and 0.13 at 900 A. It was found that the effect of aging increased strongly with decreasing wavelength. Exposure to air reduced the reflectivity of good films at 2,200 A from 0.92 to 0.90 in 1 month and to 0.88 in 1 year, at 1,600 A from 0.78 to 0.68 in 1 month and to 0.65 in 1 year, at 1,200 A from 0.48 to 0.27 in 1 month and to 0.22 in 1 year, and at 900 A from 0.20 to 0.08 in 1 month and to 0.04 in 1 year. The decrease is attributed mainly to formation of the oxide layer.

Films stored in a dessicator for several months showed somewhat higher reflectivity than those kept in normal air at a humidity of 30 to 50 percent (ref. 11). Films exposed for 20 hours to ultraviolet light lost about 0.1 in reflectivity. (For example, at 900 A, R was reduced from about 0.15 to about 0.05.)

Despite the low reflectivity after aging, the transmission is probably quite low because of the high absorptivity. In reference 12 the transmissivity of an unaged film 1,350 A thick is given as about 0.001 at wavelengths between 1,250 A and 833 A. Thus, for an aged film nearly twice as thick, less than 0.001 of the incident radiation would be expected to be transmitted through the aluminum to the plastic.

Ultraviolet Region Between 900 A and 300 A

Data for this region are few. Reference 12 shows that the transmissivity of a film increases rapidly from about 0 at 833 A to large values at 500 A. A 500-A-thick film had a transmissivity of 0.01 at 833 A and of 0.50 at 500 A. A 1,350-A-thick film had a transmissivity of 0.001 at 833 A and of 0.33 at 500 A. The reflectivity was zero between 700 A and 500 A. The absorption coefficient at 500 A was about 10^5 cm^{-1}. The intensity transmitted at this wavelength by a film 2,200 A thick is then

$$I = I_0 e^{-\alpha t} = 0.11 I_0$$

where, by virtue of zero reflectivity, I_0 is the intensity of the incident radiation.

Apparently, then, practically no ultraviolet radiation between 4,000 A and about 833 A reaches the plastic, but at a wavelength of about 500 A a considerable fraction, about 10 percent, of the incident radiation penetrates to the plastic. At this wavelength, the intensity of the sun's radiation is not yet known with certainty, but the data of reference 13 would seem to indicate that it is probably about 10^{-4}, or less, of the sun's peak intensity in the ultraviolet region, which occurs near the upper edge of the ultraviolet region, 4,000 A. The effect of prolonged exposure of the plastic to about one-tenth of this intensity in this wavelength range is not known but may well be negligible.

EQUILIBRIUM TEMPERATURES

The maximum and the minimum allowable temperatures of the skin are set by the properties of the plastic. The temperatures of the portions of the sphere that assume the extreme values are therefore of most interest. The necessity of preventing the plastic from melting can easily be shown. If the satellite should spin, there would be a centrifugal force. If the satellite should retain the inflating gas, there would be a gas pressure. Even in the absence of these forces there is another reason why melting must be prevented, and that is surface tension. What happened when a region on a piece of the satellite material was heated to melting temperatures by radiation is shown in figure 2. Surface tension cleared the region of plastic and aluminum. Two simplifications that are quite justified make the calculations of the equilibrium temperatures of the plastic easier than they otherwise would be. On account of the very small skin thickness the transfer of heat from the hotter to the colder regions of the sphere by conduction is negligible, and on account of the small heat capacity of the satellite the time required to approach thermal equilibrium closely is small compared with a half-period of revolution.

The satellite receives radiation from three sources: direct radiation from the sun, sun's radiation reflected from the earth, and direct radiation from the earth.

Direct Radiation From Sun

The solar constant C_S is the amount of radiant energy from the sun that falls on unit area at normal incidence in unit time at a location just above the earth's atmosphere at the average sun-to-earth distance. Of this flux, a fraction given by A_S is absorbed. (The fact that A_S may be a function of angle of incidence is not taken into account herein.) Therefore, the rate at which energy is received directly from the sun by the satellite is $\frac{\pi}{4} D^2 C_S A_S$.

The Smithsonian value of the solar constant is 1.946 cal/cm^2-min (ref. 14). A more recent and perhaps more accurate value is 2.00±0.04 cal/cm^2-min (ref. 15). The annual variation of incident flux between a maximum in December and a minimum in June is an additional 0.07 cal/cm^2-min. The value used herein for the flux is the mean, $C_S = 2.00$ cal/cm^2-min.

Solar Radiation Reflected From Earth

If the earth were a specular reflector of sunlight, the calculation of the rate of reception by the satellite of solar radiation reflected from the earth would be relatively simple. The reflection, however, probably occurs principally from clouds and is therefore diffuse rather than specular. Russell (ref. 16) derived an expression for the diffuse case that is applicable when the receiving body is at a distance from the reflector that is large compared with the radius of the reflector. The same expression is also derived in reference 17. The results are not applicable when the receiver is close to the reflector. Reference 18 treats the case when the satellite is less than several thousand miles from the earth and gives an expression that is, through error, just one-half that obtained herein as an approximate expression.

The energy absorbed per unit time per unit area normal to the direction of propagation, by reflection from an element of area dS on the earth is, by Lambert's law and figure 3,

$$\int \frac{C_S A_S a \cos\theta \cos\phi}{\pi l^2} dS$$

The element of area dS on the earth is

$$dS = 2\pi r_E^2 \sin\theta \, d\theta$$

From the cosine law, the distance l is given by

$$l^2 = r_E^2 + (r_E + h)^2 - 2r_E(r_E + h)\cos\theta$$

or

$$l^2 = (r_E + h)^2(1 + k^2 - 2k \cos\theta)$$

where

$$k = \frac{r_E}{r_E + h}$$

From the sine law,

$$\cos\phi = \left[1 - \frac{1}{\csc^2\theta(\cos\theta - k)^2 + 1}\right]^{1/2} = \frac{\cos\theta - k}{(1 + k^2 - 2k\cos\theta)^{1/2}}$$

The projected normal area of the satellite is $\frac{\pi}{4} D^2$. Since the radiation is solar (under the assumption of nonselective reflection), the absorptivity of the satellite for it is taken as A_S. Then the flux absorbed by the satellite is

$$\frac{\pi}{4} D^2 C_S A_S a 2k^2 \int_{\theta=0}^{\theta=\cos^{-1}k} \frac{(\cos\theta - k)\cos\theta \sin\theta}{\left(1 + k^2 - 2k\cos\theta\right)^{3/2}} d\theta$$

which has the value

$$\frac{\pi}{4} D^2 C_S A_S a \frac{4}{3k}\left[1 + \frac{k^3}{2} - \left(1 + \frac{k^2}{2}\right)\left(1 - k^2\right)^{1/2}\right]$$

It can readily be seen that, for $k \ll 1$, the factor within the brackets reduces approximately to only the term containing k^3. The expression for the flux absorbed by the satellite then agrees with that derived in reference 17 when the sun, the satellite, and the earth are in a straight line.

For the present calculations, it is convenient to make the approximation

$$\frac{4}{3k}\left[1 + \frac{k^3}{2} - \left(1 + \frac{k^2}{2}\right)\left(1 - k^2\right)^{1/2}\right] \approx 2\left[1 - \left(1 - k^2\right)^{1/2}\right]$$

inasmuch as a similar expression is obtained in the succeeding section in the derivation of the direct radiation from the earth. That the approximation is a good one for values of k not too small is shown by comparison of the two curves in figure 4. Therefore, the satellite receives from the earth as diffusely reflected sun's radiation

$$\frac{\pi}{4} D^2 C_S A_S a 2\left[1 - \left(1 - k^2\right)^{1/2}\right] \tag{1}$$

Direct Radiation From Earth

Since the earth remains at nearly the same temperature for centuries and since the outward flow of energy from the interior is very small and the net energy used by plants is very small, practically all the energy the earth receives from the sun is either reflected or radiated. The surface area of a sphere is four times the projected area. The radiation from the earth is therefore $\frac{C_S(1 - a)}{4}$. The flux falling on and absorbed by the satellite is, by Lambert's law and figure 3,

$$C_S A_E \frac{1 - a}{4} \frac{\pi}{4} D^2 \int_{\theta=0}^{\theta=\cos^{-1}k} \frac{\cos \phi}{\pi l^2} dS$$

When the same substitutions as before are made and integration is performed, the flux becomes

$$\frac{\pi}{4} D^2 C_S A_E \frac{1 - a}{2}\left[1 - \left(1 - k^2\right)^{1/2}\right] \tag{2}$$

The factor within the brackets is exact and is not an approximation as it is in equation (1).

For the conditions considered in this paper, the contributions from the three sources of energy - direct radiation from the sun, solar radiation reflected from the earth, and direct radiation from the earth - are in the ratio 100:30:4.

The total rate of energy reception is

$$\frac{\pi}{4} D^2 C_S A_S \left\{1 + 2\left(a + \frac{1 - a}{4} \frac{A_E}{A_S}\right)\left[1 - \left(1 - k^2\right)^{1/2}\right]\right\} \tag{3}$$

The total rate of energy loss is $\pi D^2 \epsilon_o \sigma \overline{T^4}$ cal/min where $\overline{T^4}$ is the fourth power of the skin temperature averaged over the surface in such a way that this expression is the rate of radiation from the satellite. At equilibrium, rates of energy reception and loss are equal:

$$\epsilon_o \sigma \overline{T^4} = \frac{1}{4} C_S A_S \left\{1 + 2\left(a + \frac{1 - a}{4} \frac{A_E}{A_S}\right)\left[1 - \left(1 - k^2\right)^{1/2}\right]\right\} \tag{4}$$

The result of the preceding analysis, equation (4), is used first to derive an expression for the equilibrium temperature of the hottest spot T_{HS} on the satellite when the satellite is so located that this temperature is a maximum. Because of the role played by the high emissivity of the inside surface of the sphere in distributing received radiation over the surface of the sphere, the condition of maximum value of T_{HS} occurs when the rate of energy reception is greatest; that is, when the satellite is directly between the sun and the earth (fig. 5). Equation (4), in fact, applies only when the satellite is so located. For unit area of the hottest spot facing the sun, the thermal balance is given by the equation

Reception from sun + Reception from inside satellite

= Radiation from both sides

The rate of reception from the sun is $C_S A_S$. The rate of radiation from both sides is $(\epsilon_i + \epsilon_o)\sigma T_{HS}^4$. The rate of reception from the inside of the satellite must next be found. Because the skin of the satellite is so thin that no significant temperature difference can exist between the outside and the inside surfaces, equation (4) can be multiplied by ϵ_i/ϵ_o and the surface area πD^2 to obtain the total rate of emission of radiation on the inside of the sphere:

$$\pi D^2 \epsilon_i \sigma \overline{T^4} = \frac{\pi}{4} D^2 \frac{\epsilon_i}{\epsilon_o} C_S A_S \left\{ 1 + 2\left(a + \frac{1 - a}{4} \frac{A_E}{A_S}\right)\left[1 - \left(1 - k^2\right)^{1/2}\right]\right\} \tag{5}$$

It is easy to show, however, that this flux is received equally by all elements of area on the inside surface of the sphere.

Any two elements of area on the surface of a sphere lie on a great circle's plane through the sphere's center (fig. 6). The intensity of the radiation from one of these elements is, by Lambert's law, proportional to $\cos \phi$. The normal projection of the receiving element is proportional to $\cos \theta$. The intensity also varies inversely as the square of the distance l between the elements. But in a circle, $l = D \cos \theta = D \cos \phi$. Therefore, the flux received by an element from another element does not depend on θ or ϕ and thus is not a function of latitude or azimuth. In other words, a given element of area contributes equally to the flux received by all other elements. Therefore, all elements receive energy at the same rate, no matter what the temperature distribution is. This result is, of course, true only for a sphere.

Therefore, the rate of reception of energy by unit area of the hottest spot from internal radiation is $\epsilon_i \sigma \overline{T^4}$ as given by equation (5). Then, in equilibrium, the thermal-balance equation for unit area of the hottest spot is

$$C_S A_S + \frac{1}{4}\frac{\epsilon_i}{\epsilon_o} C_S A_S \left\{1 + 2\left(a + \frac{1 - a}{4}\frac{A_E}{A_S}\right)\left[1 - \left(1 - k^2\right)^{1/2}\right]\right\} = \left(\epsilon_i + \epsilon_o\right)\sigma T_{HS}{}^4 \tag{6}$$

An expression is needed also for the temperature of the coldest spot T_{CS} on the satellite when the satellite is in the location that T_{CS} is a minimum - that is, when the satellite is in the earth's shadow. It then receives no direct or reflected energy from the sun, only direct radiation from the earth. From equation (4) the thermal-balance equation for the satellite when it is in the earth's shadow is

$$\epsilon_o \sigma \overline{T^4} = \frac{1}{4} C_S A_S \left(\frac{1 - a}{2}\right)\frac{A_E}{A_S}\left[1 - \left(1 - k^2\right)^{1/2}\right] \tag{7}$$

The coldest spot receives energy only by radiation from the inside surface of the satellite, at the rate

$$\epsilon_i \sigma \overline{T^4} = \frac{1}{4}\frac{\epsilon_i}{\epsilon_o} C_S A_S \left(\frac{1 - a}{2}\right)\frac{A_E}{A_S}\left[1 - \left(1 - k^2\right)^{1/2}\right] \tag{8}$$

Thermal balance of the coldest spot of the satellite is therefore given by

$$\frac{1}{4}\frac{\epsilon_i}{\epsilon_o} C_S A_S \left(\frac{1 - a}{2}\right)\frac{A_E}{A_S}\left[1 - \left(1 - k^2\right)^{1/2}\right] = \left(\epsilon_i + \epsilon_o\right)\sigma T_{CS}{}^4 \tag{9}$$

Equation (9) is based on equation (2) which contains the factor $C_S(1 - a)/4$ as the rate of radiation emitted by unit area of the earth. This factor is an average value obtained, in effect, by averaging over the entire surface of the earth, over all angles of emission, and over periods of time very long compared with a period of revolution of the satellite. Of more pertinency than the average value of T_{CS} that is given by equation (9) as a consequence of this averaging would be the minimum value of T_{CS} that could result from any possible combination of conditions that affect rate of radiation from the earth and its atmosphere. The authors have not, however, obtained quantitative results for this case, partly because of the complication and partly because possible limits of many of the factors apparently are not known. Quantitative results would require knowledge of the possible limits of vertical distribution of content of carbon dioxide, ozone, and water in its vapor, liquid, and solid phases and would require consideration of the distribution, the emissivity (as function of wavelength), and the temperature of these constituents and of the minimum possible radiation from that portion of the earth's surface that can be totally in shadow, and of the interplay of all these factors.

The final step is to substitute values for the various constants into equations (6) and (9) and to solve for T_{HS} and T_{CS}. The value of A_S for the satellite material is $1 - R_S$ or 0.10. The quantity A_E can be assumed to be the same as ϵ_o. The value of ϵ_o was found to be 0.03 and ϵ_i was found to lie within the range from 0.31 to 0.41. In the interest of conservatism, the lowest of these values was used in calculating the temperatures of the hot spot and the cold spot. Actually, use of the highest value would cause the calculated temperature to be lower by only 3° C or less. The earth's albedo has been under investigation by Danjon (ref. 19) since 1926 by means of measurements of the intensity of the earthshine reflected by the moon. Fritz (ref. 20) has computed the earth's albedo from measurements and estimates of the individual albedos of clouds, sea, and so forth. Both investigators agree that the average value is about 0.36. There is evidence of a minimum value of albedo of about 0.32 in July and a maximum value of 0.52 in October for the entire earth. Further, one can consider also the case when the portion of the earth seen by the satellite is completely covered with the type of cloud that has the highest albedo, 0.85 (ref. 21).

Calculations of the satellite's highest temperature were made by use of equation (6) for three values of albedo (the fact that different values can affect the direct radiation can be neglected) and for perigee altitudes of 800 and 1,000 miles, for which k is, respectively, 0.833 and 0.80. The temperature of the coldest spot on the satellite was calculated by use of equation (9) for the average value of the albedo and the two altitudes. The results are shown in table I, which follows:

TABLE I

TEMPERATURES OF HOTTEST AND COLDEST SPOTS ON SATELLITE

Altitude, miles	Earth's albedo, a	T_{HS}, °C	$\left(\overline{T^4}\right)^{1/4}$, °C	T_{CS}, °C	$\left(\overline{T^4}\right)^{1/4}$, °C
800	0.36	153	137	-105	-100
800	.52	161	146		
800	.85	176	164		
1,000	.36	150	133	-109	-105
1,000	.52	158	142		
1,000	.85	171	159		

These calculated maximum and minimum values of the temperature are believed to be acceptable. Limited tests have shown that no visible changes occur in the aluminized plastic when heated to 230° C in an inert gas, although the tensile strength of the plastic is zero at 245° C. Tests have also shown that seams such as those which may be used in constructing the sphere are satisfactory at -100° C so far as adhesion properties and flexibility are concerned.

Comparison of the temperature of the hottest spot with the mean temperature of the satellite when it is located directly between the sun and the earth and comparison of the temperature of the coldest spot with the mean temperature of the satellite when it is located in the shadow of the earth are of interest. The analysis used herein does not, however, lead to values of the arithmetic mean of the satellite temperature but results in only the fourth root of the mean of the fourth power of the temperature $\left(\overline{T^4}\right)^{1/4}$ from equation (4) for the sunlit case and from equation (7) for the shadow case. This quantity is of course weighted in favor of the larger values. The comparisons are made in table I. The extreme temperatures are seen to be rather close to the weighted mean temperatures.

If, finally, because of strength or other considerations, 1/2-mil-thick Mylar should be chosen rather than the 1/4-mil-thick Mylar considered herein, the weight of the plastic would be doubled but the radiation characteristics of the satellite would not be significantly affected. The emissivity of the thicker plastic would be expected to be somewhat greater than that of the thinner plastic, but the calculated temperatures would be changed by only a very few degrees.

CONCLUDING REMARK

The design study of the characteristics, with respect to radiation, of a 100-foot-diameter hollow spherical satellite formed of 1/4-mil-thick Mylar and coated with a 2,200-angstrom-thick film of vapor-deposited aluminum has predicted that the satellite performance will be satisfactory with regard to visibility, reflectivity for radar waves, maximum and minimum temperatures, and probably will be satisfactory with regard to resistance to deterioration by solar ultraviolet radiation.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Field, Va., July 24, 1959.

REFERENCES

1. Holland, L.: Vacuum Deposition of Thin Films. John Wiley & Sons, Inc., 1958.

2. Holland, L.: The Effect of Vapor Incidence on the Structure of Evaporated Aluminum Films. Jour. Optical Soc. of America, vol. 43, no. 5, May 1953, pp. 376-380.

3. Tousey, R.: Optical Problems of the Satellite. Jour. Optical Soc. of America, vol. 47, no. 4, Apr. 1957, pp. 261-267.

4. Russell, Henry Norris, Dugan, Raymond Smith, and Stewart, John Quincy: Astronomy. II - Astrophysics and Stellar Astronomy. Ginn and Co., c.1955.

5. Abraham, Max, and Becker, Richard (John Dougall, trans.): The Classical Theory of Electricity and Magnetism. Blackie & Son, Ltd. (London and Glasgow), 1937, p. 192.

6. Purcell, E. M.: The Radar Equation. Radar System Engineering, ch. 2, Louis N. Ridenour, ed., McGraw-Hill Book Co., Inc., 1947, p. 21.

7. Pierce, J. R.: Orbital Radio Relays. Jet Propulsion, vol. 25, no. 4, Apr. 1955, pp. 153-157.

8. Hass, Georg: Filmed Surfaces for Reflecting Optics. Jour. Optical Soc. of America, vol. 45, no. 11, Nov. 1955, pp. 945-952.

9. Schulz, L. G.: An Interferometric Method for the Determination of the Absorption Coefficients of Metals, With Results for Silver and Aluminum. Jour. Optical Soc. of America, vol. 41, no. 12, Dec. 1951, pp. 1047-1051.

10. Hass, G., Hunter, W. R., and Tousey, R.: Reflectance of Evaporated Aluminum in the Vacuum Ultraviolet. Jour. Optical Soc. of America, vol. 46, no. 12, Dec. 1956, pp. 1009-1012.

11. Hass, G., Hunter, W. R., and Tousey, R.: Influence of Purity, Substrate Temperature, and Aging Conditions on the Extreme Ultraviolet Reflectance of Evaporated Aluminum. Jour. Optical Soc. of America, vol. 47, no. 12, Dec. 1957, pp. 1070-1073.

12. Walker, W. C., Samson, J. A. R., and Rustgi, O. P.: Optical Properties of Al for $h\nu$ Between 10 and 26 ev. Jour. Optical Soc. of America (Letters to the Editor), vol. 48, no. 1, Jan. 1958, p. 71.

13. Byram, E. T., Chubb, T., and Friedman, H.: The Study of Extreme Ultraviolet Radiation From the Sun With Rocket-Borne Photon Counters. Rocket Exploration of the Upper Atmosphere, R. L. F. Boyd and M. J. Seaton, eds., Interscience Publ., Inc. (New York), 1954, pp. 276-278.

14. Forsythe, William Elmer: Smithsonian Physical Tables. Ninth rev. ed., Smithsonian Misc. Coll., vol. 120, 1956, p. 720.

15. Johnson, F. S., Purcell, J. D., Tousey, R., and Wilson, N.: The Ultraviolet Spectrum of the Sun. Rocket Exploration of the Upper Atmosphere, R. L. F. Boyd and M. J. Seaton, eds., Interscience Publ., Inc. (New York), 1954, p. 288.

16. Russell, Henry Norris: On the Albedo of the Planets and Their Satellites. The Astrophysical Jour., vol. XLIII, no. 3, Apr. 1916, pp. 173-195.

17. Zirker, J. B., Whipple, F. L., and Davis, R. J.: Time Available for the Optical Observation of An Earth Satellite. Scientific Uses of Earth Satellites, James A. Van Allen, ed., Univ. of Michigan Press (Ann Arbor), c.1956, pp. 23-28.

18. Gast, P. R.: Insolation of the Upper Atmosphere and of a Satellite. Scientific Uses of Earth Satellites, James A. Van Allen, ed., Univ. of Michigan Press (Ann Arbor), c.1956, pp. 73-84.

19. Danjon, André: Albedo, Color, and Polarization of the Earth. The Earth as a Planet, Gerard P. Kuiper, ed., The Univ. Chicago Press, c.1954, pp. 726-738.

20. Fritz, Sigmund: The Albedo of the Planet Earth and of Clouds. Jour. Meteorology, vol. 6, no. 4, Aug. 1949, pp. 277-282.

21. Anon.: American Institute of Physics Handbook. McGraw-Hill Book Co., 1957, p. 2 - 132.

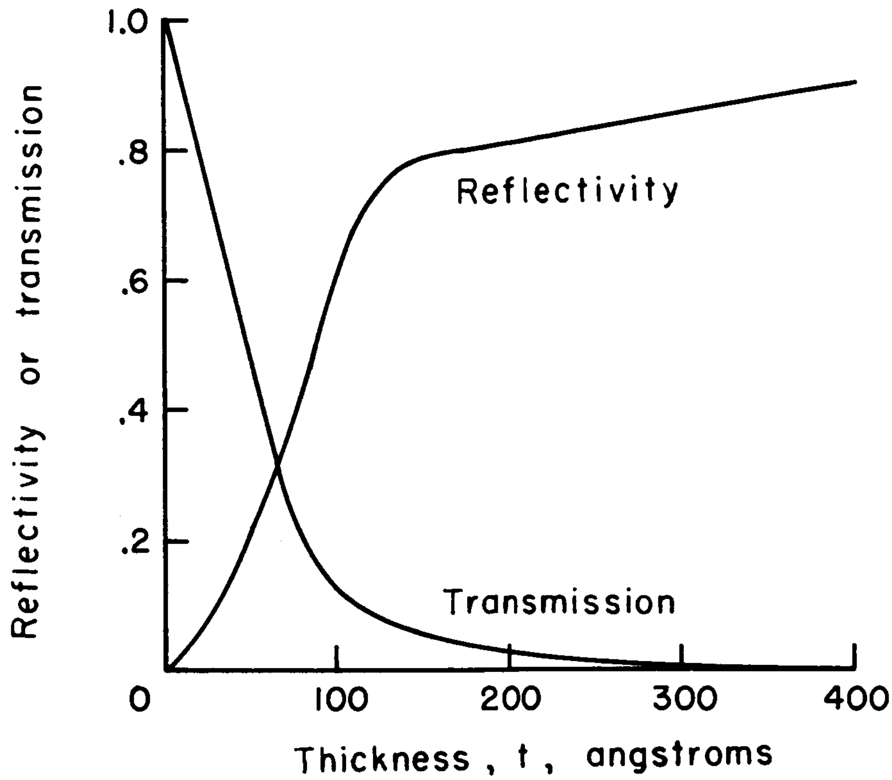

Figure 1.- Reflectivity and transmission at 4,600 A of aluminum films as functions of thickness. (Data are from ref. 2.)

L-59-1859

Figure 2.- Hole caused by surface tension when a sample of the aluminized plastic was heated and melted locally.

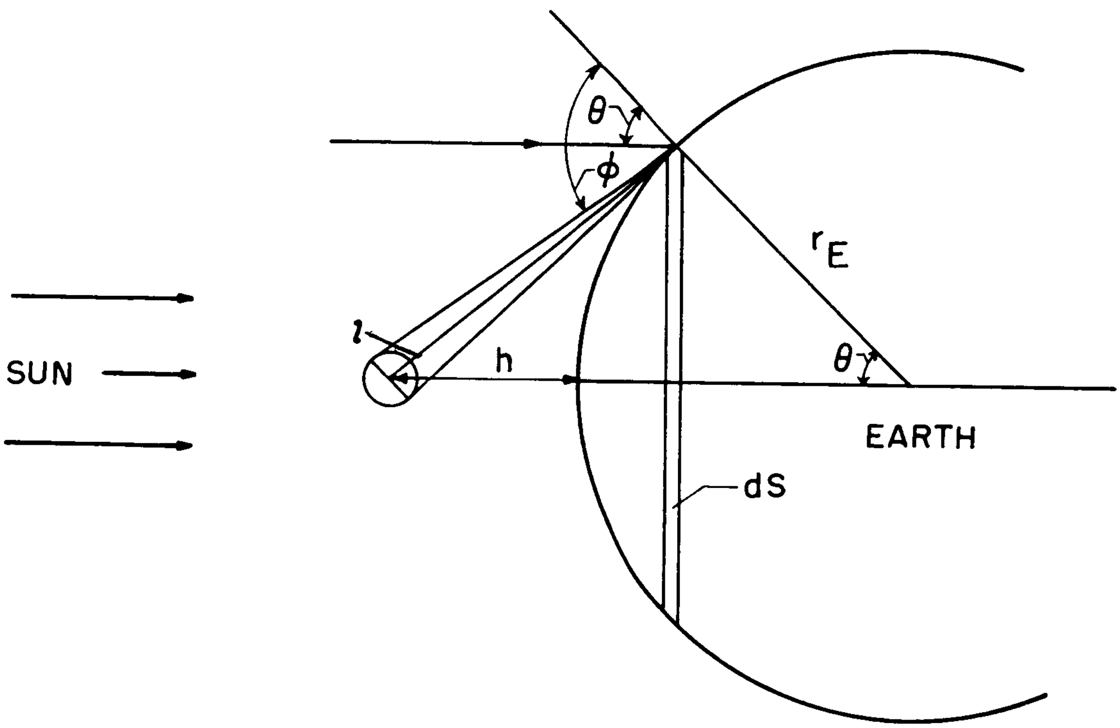

Figure 3.- Sketch used in deriving expressions for flux received by satellite.

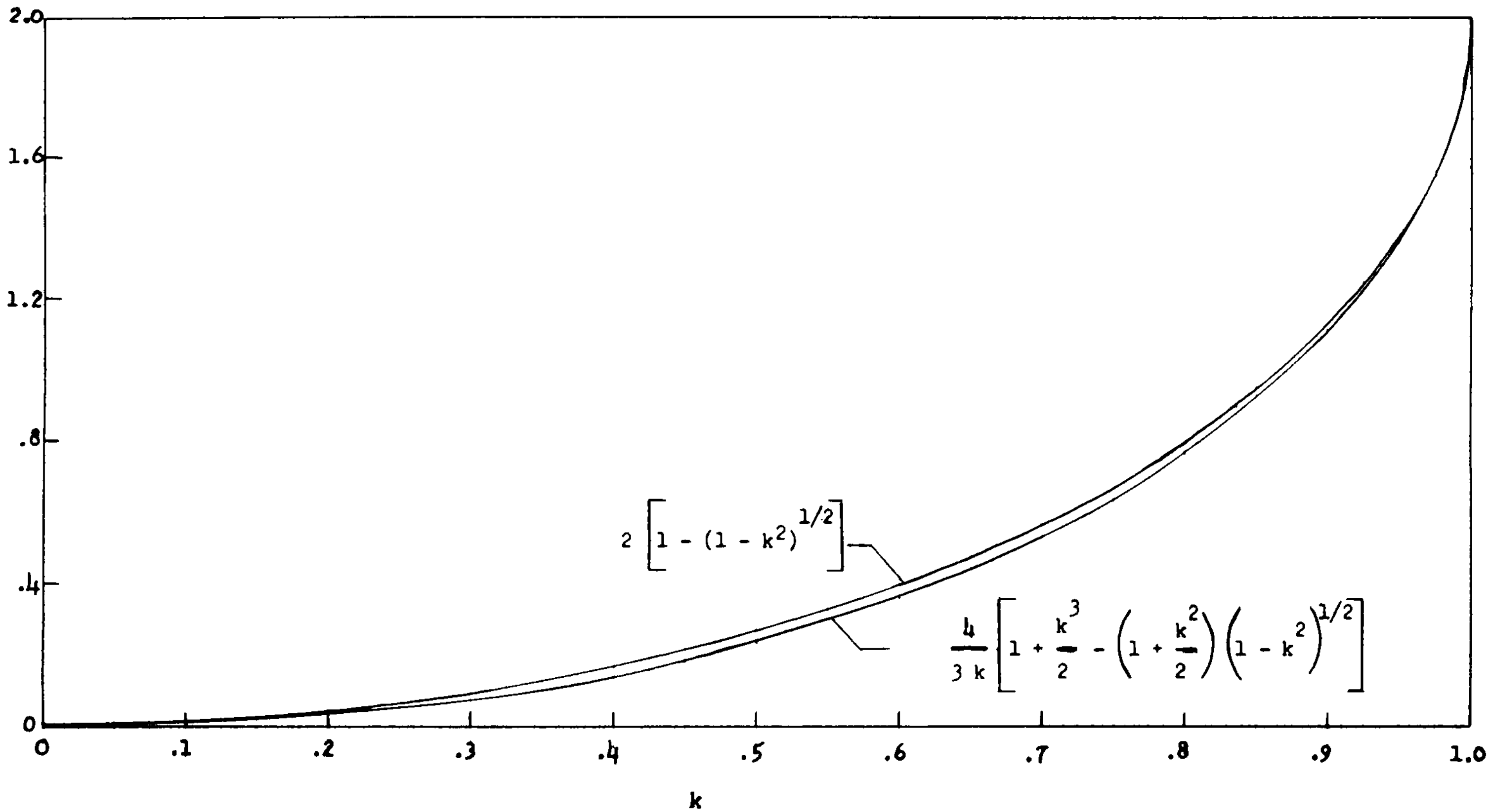

Figure 4.- Comparison of exact and approximate expressions.

4Z

L-252

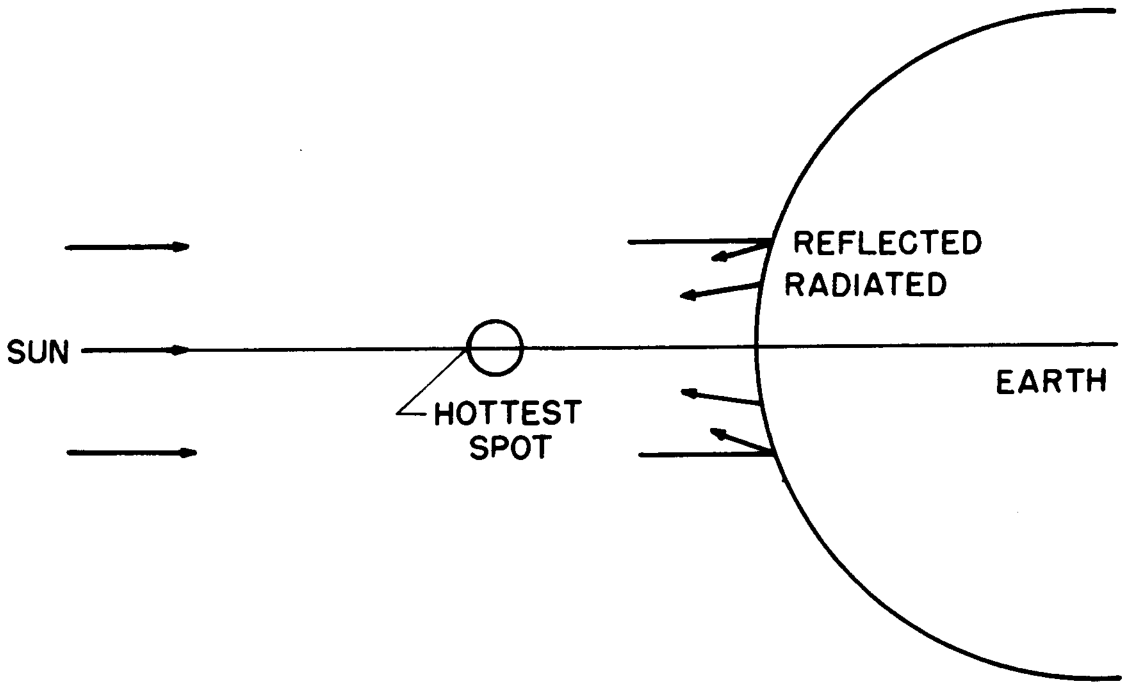

Figure 5.- Location of hottest spot on satellite.

L-252

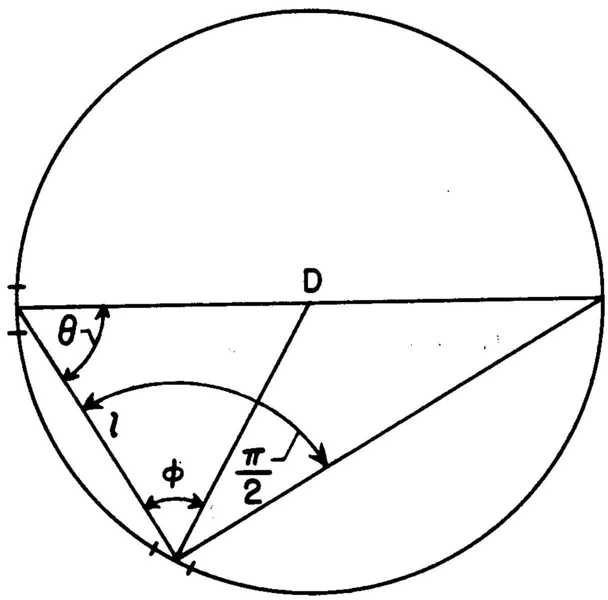

Figure 6.- Geometrical relations in a circle.